Basic Information:
Course Title: Symmetries, Supersymmatries, Deformed Symmetries and Models of Interacting Many-Body Systems
Lecturer: Prof. DSc Boyka Aneva
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Total Teaching Hours: 30

Annotation (up to 150 words)

Group theory and their representations provide a natural mathematical framework for describing symmetries in nature. The group-theoretical approach has become extremely important in every field of modern physics, serving as a tool for solving physical problems involving space-time symmetries, internal symmetries, and discrete symmetries. Supersymmetry is a newer type of symmetry whose transformations link fermions and bosons, objects with different spin and statistics. Its ideas first appeared in dual models in scattering theory, which, in fact, led to the discovery of superstrings. The higher symmetry leads to cancellation of divergencies and allows for the formulation of renormalizable models of elementary particle interactions. Therefore, supersymmetry provides a natural framework for unifying and describing all particles and their interactions.

Quantum deformed symmetries based on noncommutative spacetime algebraic structures have gained particular significance. Emerging from the inverse problem method and developed to construct exact solutions for quantum integrable systems, these rich mathematical structures find applications in fields of physics and mathematics that seem, at first glance, to be far removed from one another. Quantum groups are the mathematical language of topological quantum computing, where their application focuses on braiding operations and information-storing qubits for the construction of fault-tolerant quantum computers.

Course content (brief description by topics or modules)
Topic / Module 1: Symmetries, their representations and applications to physics
Topic / Module 2: Supersymmetries and their representations, supersymmetric models
Topic / Module 3: Deformed symmetries, quantum groups and applications

Teaching and assessment methods
Lectures and consultations, assessment based on an interview or essay

Competencies acquired as a result of training (3–5 points)

1. In-depth knowledge of contemporary fields in quantum physics, statistical physics, and high-energy physics
2. Expertise in the modern mathematical tools of representation theory (Lie algebras, superalgebras, quantum groups), differential geometry, topology, and tensor analysis.
3. Critical thinking and initiative: evaluation of existing models and creation of theoretical models of many-body systems
4. Independent use of the literature and skills for independent preparation of scientific research

5. Presentation skills, preparation of a manuscript for a scientific journal

Literature:

1. E. Wigner, Group Theory and its Application to the Quantum Mechanics of Atomic Spectra, Elsevier, 2012
2. M. Hamermesh, Group Theory and its Application tp Physical Problems, Addison-Wessley Publ., 1962
3. G. Lyubarsky, Theory of Groups, Pergamon 2013
4. M. A. Naimark, Theory of Representations, Springer Verlag NY, 1982
5. M. A. Naimark, Linear Representations of the Lorentz Group, Pergamon, 1964
6. I. M. Gel'fand et al., Representations of the Rotation and Lorentz Groups and their Applications, Pergamon, 1963
7. N. Bogolyubov, A. Logunov, I. Todorov, Chapter 2 in the book Axiomatic Approach in Quantum Field Theory, W. A. Benjamin, Advanced Book Program, 1975
8. Yu. V. Novozhilov, Introduction to Elementary Particle Theory, Pergamon, 1975
9. A. Rumer, A. Fet, Theory of Unitary Symmetry, (in Russian) Nauka, Moscow, 1978
10. E.Wigner, Unitary representations of the inhomogeneous Lorenz Group, Ann.Math.40, 149 (1939)
11. N. N. Bogoliubov and D. V. Shirkov, Introduction to the Theory of Quantized Fields, Wiley-Interscience, 1980
12. F. Berezin, The Method of Secondary Quantization, Nauka, Moscow, 1965
13. J.Wess, J.Bagger, Supersymmetry and supergravity, Princeton Univ.Press, 1983
14. R.Haag, J.Lopuszanski,M.Sohnius, Nucl.Phys. B88, 257, (1975)
15. S.Gates, M.Grisaru,M.Rocek,W.Siegel, One thousand and one lessons in supersymmetry, Benj.Read.,Mass. 1983
16. F.Guersy, Particles and Gravity, World Sci., Singapore , 1984
17. J.Wess, B.Zumino, Nucl.Phys.B70, p.39, Nucl.Phys.B78, p1, (1974)
18. J.Wess, B.Zumino, Phys.Lett.49B, p.52, (1974); Phys.Lett.66B, p.361, (1977)
19. E.Witten, Nucl.Phys.B186, p.412, (1977)
20. J.Polchinski, Superstring theory and beyond, Cambridge Univ.Press, 1998
21. H.Haber, The status of the minimal supersymmetric standard model and beyond, hep-ph/9709450
22. W.Hollik et al, Renormalization of the minimal supersymmetric standard model, hep-ph/0204350
23. D.Miller, R.Nevrozov, P.Zorwas, The next-to-minimal supersymmetric standard model, hep-ph/0304049
24. G.Senjanovic et al, The minimal supersymmetric unified theory, hep-ph/0306242, hep-ph/0402122
25. J.Fuchs, C.Schweigert, Symmetries, lie algebras and representstions, Cambridge, 1997
26. Y.Manin, Quantum Groups and noncommutative geometry, Princeton Univ.Press, 1991
27. R.Baxter, Exactly solved models in statistical mechanics, Acad.Press New York, 1982
28. Quantum Computing for High Energy Physics: State of the Art and Challenges, PRX Quantum 5, 037001 (2024)
29. Z. Wang, Quantum Computing: A quantum Group Approach, math.QA/1301.4612

Additional information (optional) (e.g., special requirements, laboratory equipment, prior knowledge)

This course aims to broaden and deepen students' knowledge of Lie group theory, superalgebras and their representations, Hopf algebras, and Hecke algebras, and to focus on their applications to exactly solvable models of quantum interacting systems and quantum computing.

Program: Main topics

1. Symmetries and conservation laws. From symmetries to supersymmetries. Noncommutative geometry and physical models.
2. Finite groups. Representations of the symmetric group. Young Tableaux.
3. Topological groups. Lie groups. Group manifolds. Vector fields. Maurer-Cartan theory.
4. Affine algebras. Katz-Moody algebras.
5. Unitary symmetries. Representations. Applications in particle physics.
6. Supersymmetries. Clifford algebras. Spinor representations.
7. Poincaré and conformal supersymmetry. Supersymmetric models.
8. Supersymmetric gauge models.
9. Grand Unification Theories. Minimal Supersymmetric Standard Model.
10. Supersymmetric phenomenology.
11. Hopf algebras. Deformed universal-enveloping algebras. Quantum groups
12. Noncommutative space. Differential calculus. Deformed Heisenberg algebra
13. Noncommutative gauge models and connection to string models.
14. Hecke algebras. Young–Baxter equation. Integrable models and quantum groups
15. Quantum groups and quantum computing.

The course is intended for doctoral students in physics from the following fields:

- Theoretical and Mathematical Physics 01.03.01,
- Nuclear Physics 01.03.04,
- Particle Physics 01.03.05,
- condensed matter physics 01.03.25, 01.03.26,
- astrophysics 01.04.02

Program prepared by:

Prof. DSc B. Aneva

19.05.2026